When the pressures on the floor of the cavity are related to the pressure in the separation corner (Fig. 1), the distributions of the ratio are found to vary with cavity depth to length ratio as in uniform flow. However, the mean pressure level in the cavity depends on the degree of acceleration of the freestream and can be substantially lower than in uniform flow. The data are shown on Fig. 1. Figure 2 shows a schlieren picture of the shear layer over the notch. Indeed, the shear layer dips into the cavity upon separation with an attendant expansion.

If we assume that the perturbation of the freestream because of the cavity is small and the flow is isentropic, we can express the characteristic length λ^{-1} in terms of the freestream Mach number gradient

$$\lambda^{-1} = \gamma M_0\{1 + [(\gamma - 1)/2]M_0^2\}^{(1-\gamma)/(\gamma-1)} dM/dx \quad (2)$$

where M_0 is to be taken as the Mach number over the boundary immediately ahead of separation. The deflection of the separation streamline into the cavity at separation is given by

$$-\frac{d\theta}{dM} \cong \frac{1}{M_0}(M_0^2 - 1)^{1/2}\bigg/\left(1 + \frac{\gamma - 1}{2} M_0^2\right) \quad (3)$$

Combining, the shape of the separating streamline (origin of coordinates at the separation corner) is found

$$-y/\lambda = [(M_0^2 - 1)^{1/2}/2\gamma M_0^2](x/\lambda)^2 + \theta_s(x/\lambda) \quad (4)$$

where $(-\theta_s)$ is the sudden (Prandtl-Meyer) deflection into the cavity at the separation corner. Imposing a "closure condition" that the separation-streamline reattaches to the downstream edge of the notch allows a solution for θ_s and hence for the pressure inside the cavity (identified with the pressure in the separation corner). This is plotted on Fig. 1. The correlation with measurements is good.

The existence of an externally imposed pressure gradient must obviously also affect the critical length of the notch for which the flow can no longer "jump" across it entirely. Models which do not take into account the interaction between the "dead" air in the cavity and the external stream cannot deal with the closure problem. It is observed that even in the absence of an external pressure gradient the shear layer dips progressively further into the cavity as its length is increased. In a turbulent, reasonably thin, upstream boundary layer at intermediate supersonic Mach numbers, closure occurs at

$$(L/H)_{cr} \cong 11; \quad \lambda^{-1} = 0 \quad (5)$$

Fig. 2 Schlieren photograph of the shear layer over a notch: $L/H = 4$, $\lambda = 5.77$ in., $M_0 = 1.89$.

Assuming that the inward deflection of the shear layer due to this interaction and that due to the acceleration in the freestream are simply additive, we can estimate the critical closure length of rectangular notches from

$$\left(\frac{H}{L}\right)_{cr} = \left(\frac{H}{L}\right)_{cr,\lambda^{-1}=0} + \frac{d(y/L)_{max}}{d\lambda}\lambda = 0.091 + \frac{(M_0^2 - 1)^{1/2}}{8\gamma M_0^2}\left(\frac{L}{\lambda}\right)_{cr}$$

In these experiments the $\frac{1}{2}$-in.-deep cavity collapsed at $(L/\lambda)_{cr} = 0.72$; $(L/H)_{cr} = 8.2$; $\lambda^{-1} = 0.173$ in.$^{-1}$. The previous equation yields a value of $(L/H)_{cr}$ of 8.3. It appears that the model is successful.

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Mechanism of Entrainment in Turbulent Wakes

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THE existence of an abrupt limit to the region of turbulent flow is characteristic of wakes and free shear turbulence in general. The interface between the laminar and turbulent regions is a continuous but highly convoluted surface which is responsible for the intermittently turbulent output of a fixed probe placed in the transition region. Although considerable effort has gone into understanding the nature of the flow near this interface, it is still not clear how the turbulence spreads.

Townsend[1] has suggested that wake growth is essentially a surface phenomenon. In his view the basic mechanism of wake growth is a process of small scale nibbling by the turbulence at the interface. Like a flame front the wake thus advances at an essentially uniform rate over its entire surface. In addition the turbulence gives the wake a qualitative resemblance to an elastic solid so that initially random surface irregularities are unstable. Their growth leads to an increase of the total surface area and hence an increase of the entrainment rate.

However, it may be argued that Townsend's scheme does not go far enough. He used a simple model for the turbulent wake: an elastic jelly surrounded by an inviscid freestream. To simplify the computations it was also assumed that there was no mean velocity profile within the wake and consequently no straining of the turbulence. Mobbs[2] made an experimental investigation of just such an unstrained free turbulent flow. While he found that the boundary did indeed develop deep convolutions, he also observed that there was no entrainment of laminar fluid by this "wake." The apparent growth of the wake was due to the increasing amplitude of the

Received March 8, 1971; revision received April 26, 1971.

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Index Category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.

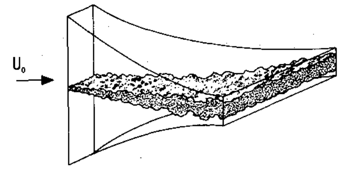

Fig. 1 The shape of the distorting duct used by Townsend, Mobbs, and Keffer; the walls are designed to provide constant rates of strain across the flow without producing any streamwise acceleration.

surface waves; there was no increase in the actual volume of turbulent fluid.

The wake subsequently entered a constant area distorting duct which stretched it in a direction perpendicular to the mean flow axis and parallel to the boundary surfaces, Fig. 1. When thus subjected to plane straining, the wake began to grow in the usual sense, entraining nonturbulent fluid. This was attributed to an increase in the anisotropy of the turbulence. However, the strain ellipsoid due to the usual Gaussian velocity profile of free shear flows does not have the same orientation as that set up by the distorting duct and so the anisotropy of the turbulence need not be the complete explanation of wake growth.

A clue as to how the straining by the duct caused entrainment may be found in Keffer's[3] study of the effect of a similar distortion on the turbulent wake of a cylinder. He found that the distortion selectively amplified those eddies which were aligned with the axis of stretch and so reduced a fully turbulent wake to a von Kármán-like double row of vortices. Much the same thing may be assumed to have occurred in Mobbs' duct. Thus it might be hypothesized that entrainment by the wake depends on the action of large rotating eddies whose presence distorts the boundary, rather than Townsend's boundary waves per se.

A visualization study undertaken by the authors tends to verify this hypothesis. The study was conducted in the wake of a sphere towed through a water filled trough at a Reynolds number of 24,000. The common technique of injecting dye behind the model to mark its wake visible was used. In addition, however, small drops of colored dye were placed in the path of the sphere. Since the "freestream" was actually still water these drops maintained their shape and identity. In this way it was possible to mark a particle of the freestream and follow its motion as it was ingested by the wake. Color movies of the mixing process were made and later studied with a stop-action, analyst type projector, Fig. 2.

It was observed that entrainment occurs in three phases. Initially, as the wake boundary approaches, a marked droplet in the freestream deforms as though it will be pushed out of the way by the oncoming boundary wave; that is, as though the wake were a solid. Then, when the wake reaches it, the droplet is swirled into the wake by the spinning motion of the large vortex associated with the surface wave. The ingested fluid is not immediately mixed throughout the wake by the turbulence, but continues to spin with the vortex that first swept it in. A filament of dye laid down parallel to the wake axis appears to coil up like a rope as it is taken into the wake. Turbulent mixing occurs in the final phase of entrainment, after the large eddy has stopped spinning.

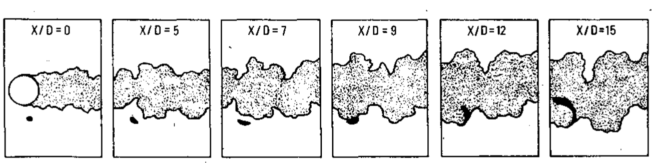

Fig. 2 Entrainment of an element of the freestream by the axisymmetric turbulent wake; tracings from 16 mm filmstrip; Re = 24,000.

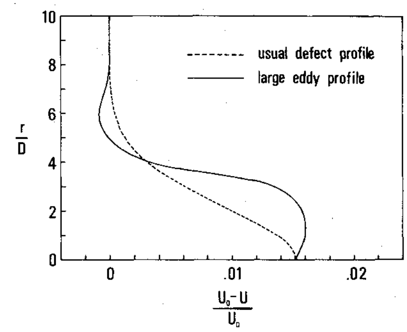

Fig. 3 The velocity profile across the large eddies found in the wake of a disk; the eddies sampled were those for which the interface between turbulent and nonturbulent fluid occurred above $r/D = 4$; $Re = 12{,}500$ and $x/D = 100$.

Further evidence for the existence of these large vortices may be found in the work of Oswald and Kibens.[4] They have utilized a method of conditioned sampling to make measurements in the intermittent region of the wake of a disk. This technique is quasi-stroboscopic, admitting a signal to the sampler only when specific conditions have been met; for example, whenever the flow is turbulent at the location of the sensor.

Using this technique, they sought to obtain mean velocity profiles across the wake boundary as it passed a detector probe which was positioned at a chosen radial station. For large values of this radial coordinate they were, in effect, obtaining average velocity profiles across the largest eddies. Such a profile is reproduced in Fig. 3 with the conventional defect profile at the same downstream station for comparison. The eddy profile strongly suggests a large vortex translating at a speed somewhat less than the freestream velocity. Also apparent is the effect of shear at the wake boundary on the external flow. It is this shearing that is responsible for the initial deformation of the dye droplets observed in the tow tank experiment. Entrainment thus begins where viscous effects become important in the irrotational freestream.

Taken together, these observations imply that the rotation of large eddies, controlling entrainment and the form of the interface, is the principal mechanism of wake growth. The problem of the origin of these eddies must now be considered. Of course, it is known that turbulent vortex shedding persists in the wake of a cylinder up to a Reynolds number of 10,000 and perhaps beyond. Indeed, it is this flow that shows the largest rate of entrainment, an order of magnitude larger than the turbulent boundary layer, for example. In the far wake of the cylinder and in jets and other flows which do not originate with a recirculation region, another mechanism clearly must be operable.

In considering the generation of ocean waves by the wind, Miles[5] has shown that shear flow over a wavy surface leads to a layer of concentrated vorticity at the critical height where the velocity in the shear profile equals the wave speed. The effect of the vortices in this layer is shown to be a transfer of momentum from the mean profile to the wave motion. It is not unreasonable to propose that a similar interaction between Townsend's surface waves and the mean shear profile of the wake itself are the origin of the large vortices.

The wake model suggested here may be summarized as follows. After an initial development length in which the turbulence intensity reaches its peak value, the characteristic similarity profiles are attained. In the region of similarity the effect of the turbulence leads to an instability of the interface and surface waves form in response to random disturbances. These waves interact with the mean velocity profile to form large vortices which entrain ambient fluid and take momentum from the mean flow before decaying. It should be noted that the difference between Townsend's "nibbling" and entrainment by the rotation of large eddies may only be a matter of scale. The swirling action of the smallest eddies is probably similar to that of the largest, although their origin is not the same.

This cycle of wave formation, vortex growth and entrainment, vortex decay and turbulent diffusion continues until the velocity defect and mean strain field are removed. At this point viscous forces have become important and the turbulence energy is dissipated as heat.

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Flowfield Produced by Trailing Vortices in the Vicinity of the Ground

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WITH the current interest in the behavior of aircraft trailing vortices it is unnecessary to emphasize the importance of studying how these are influenced by the ground during the takeoff and landing phases. The motion of the cores has been experimentally shown, for example by Dee and Nicholas,[1] to agree moderately well with the paths predicted by the simple two-dimensional theory for a pair of concentrated vortices above an infinite plane. They follow curves in the cross flow plane, of the form

$$1/y^2 + 1/z^2 = \text{const}$$

and, as Fig. 1 shows, the transition from the vertical descent to a horizontal motion is fairly abrupt. The observed flight data of Dee and Nicholas[1] show tolerable agreement with this predicted path except for a feature which was not strongly emphasized in their report. Examination of the tabulated data shows that in many tests the vortices departed from the

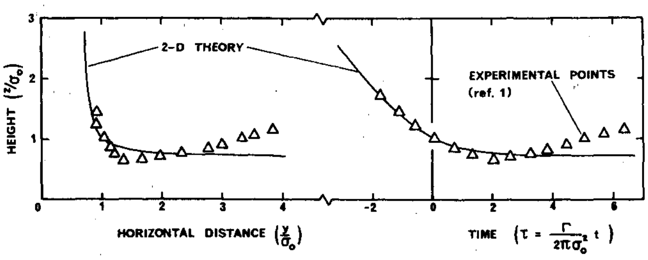

Fig. 1 Theoretical and experimental vortex paths.

Received March 16, 1971; revision received May 3, 1971.
Index Categories: Airplane and Component Aerodynamics; Aircraft and Component Wind Tunnel Testing; Jets, Wakes and Viscid-Inviscid Flow Interactions.

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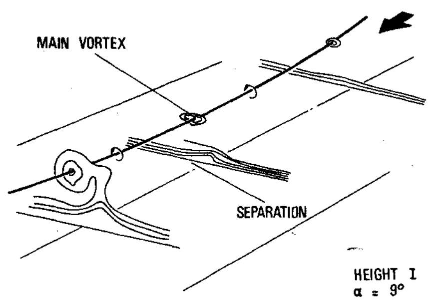

Fig. 2 Total head contours in planes across the flow downstream of the wing.

theoretical path by rising again after having descended close to the ground (Fig. 1).

To attempt to explain this phenomenon, an experiment was conducted at Imperial College to qualitatively determine the cause. A half span rectangular wing of 8 in. chord and 20 in. semispan was mounted on the wall of a 5 × 4 ft low-speed wind tunnel so that the single trailing vortex passed over a moving floor to correctly simulate the ground boundary condition. The test speed was 87 fps.

Since the primary objective was to establish the features of the flowfield that cause the vortex to rise, the investigation was centered around a series of total-head surveys in planes across the flow performed with a $\frac{1}{16}$ in. o.d. Kiel tube[2] (i.e., a yaw-insensitive total-head probe). A few additional surveys were made with a 5-tube yaw-meter to measure the strength of the trailing vortex.

Two typical total head surveys are presented as contour maps in Figs. 2 and 3. They depict what would have been the nearer of a pair of trailing vortices, and they reveal important details of the flowfield which account for the departure of vortices from the theoretical prediction. The surveys can be interpreted by recalling that the total head remains constant throughout a steady flowfield except where viscosity is active. In regions of shear (e.g., boundary layers, vortex sheets and cores) the total head falls below the freestream value. Thus at the most upstream station in Fig. 2, we can readily identify the vortex core and a small remnant of the tunnel boundary left above the moving floor because of insufficient bleed. The vortex induces a cross flow on the floor with an attendant suction peak beneath the core. Consequently, the boundary layer resulting from this cross flow has to negotiate an adverse pressure gradient once it has passed under the vortex. When the vortex is sufficiently near the ground, the pressure gradient is strong enough for separation to occur, and a bubble forms containing vorticity of opposite sense to main vortex (Fig. 4a). Progressing downstream, we find this bubble growing rapidly to the point where it detaches from the floor as a secondary vortex fed by a vortex sheet from the separation point (Fig. 4b). The development of this flow can be traced from the measured total-head surveys shown in Figs. 2 and 3.

The subsequent motion of two vortices is a complicated problem in vortex dynamics, but while the secondary core remains outboard of the main core, it will induce an upward

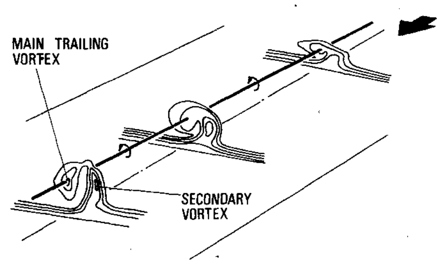

Fig. 3 Surveys of total head taken with the wing at a lower height.